UNIT 7 ARITHMATIC AND GEOMETRIC SEQUENCES

SHORT QUESTIONS

Q.1- What is general A.P and find its nth term.

Ans. General A.P is the progression $a, a+d, a+2d, a+3d...$ where a is the 1st term and d is the common difference of A.P, So

$a_1 = a$, $a_2 = a+d$, $a_3 = a+2d$, $a_4 = a+3d$ ...

These terms show that: $a_n = a+(n-1)d$

Q.2- Define and find arithmetic mean between *a* and *b*.

Ans. The number 'A' is said to be an arithmetic mean between two numbers a and b if a, A, b are in A.P, So,

$A-a = b-A =$ Common difference

$$\Rightarrow a+b = 2A \Rightarrow A = \frac{a+b}{2}$$

Q.3- 8 and 12 are two A,Ms between *a* and *b*. Find *a* and *b*.

Solution: By the given condition.

$a, 8, 12, b$ are in A P, So

$8-a = 12-8 = b-12$ = Common difference

$8-a = 4 = b-12$

$8-a = 4$ and $4 = b-12$

$a = 4$ and $b = 16$

Q.4- Define a sequence or progression.

Ans. A sequence is an arrangement of numbers written in a definite order according to some specific rule. A sequence is also called progression. For example:

(i) $1, 3, 5, 7 \ldots$ (ii) $2, 6, 10, 14 \ldots$ (iii) $3, 6, 12, 24 \ldots$

These are sequences or progressions.

Q.-5 Differentiate finite and infinite sequence.

Ans. If a sequence has its last term, it is called finite sequence.

Example:

$1, 3, 5, 7, \ldots 31$ and $2, 6, 18, 54, \ldots, 486$ are finite sequences.

If a sequence does not have its last term, it is called infinite sequence.

Example: $2, 4, 6, 8, \ldots$

and $1, 5, 9, 13, \ldots$ are infinite sequences

Q.6- Define Arithmetic Progression (A.P)

Ans. The sequence of numbers in which each term is obtained by adding a fixed number to the preceding term is called arithmetic progression.

For Example: $3, 7, 11, 15, \ldots$ is an A.P

Q.7- Define Geometric Progression (G.P)

Ans. A sequence of numbers in which each term is obtained by multiplying the preceding term by a fixed number is called a geometric progression G.P.

Example: $2, 6, 18, 54, \ldots$ is a G.P.

Q.8- Define Geometric Mean between *a* and *b*. Find its value.

Ans. A number 'G' is said to be geometric mean between a and b if a, G, b are in G.P

i.e $\frac{G}{a} = \frac{b}{G} =$ Common ratio

$\Rightarrow G^2 = ab$

$\Rightarrow G = \pm\sqrt{ab}$

$\Rightarrow$ Positive G.M $= +\sqrt{ab}$

Q.9- How many terms are there in the A.P 3, 7, 11, ...59?

Solution: Here $a=3,\ d=4,\ a_n=59, n=?$

Using formula

$a_n=a+(n-1)d$

$59=3+(n-1)(4)$

$4(n-1)=59-3$

$n-1=\dfrac{56}{4}$

$n=14+1=15$

Thus there are 15 terms in this A.P

Q.10- Find G.M between $2x^2$ and $8y^4$.

Ans. Given that $a=2x^2, b=8y^4$

G.M = ?

We have.

$$G=\sqrt{ab}$$
$$=\sqrt{2x^2\times 8y^4}=\sqrt{16x^2y^4}$$
$$G=4xy^2$$

SOLVED EXERCISES

EXERCISE 7.1

Q.1- Write the first three of the following:

(i) $a_n=n+3$ *(ii)* $a_n=(-1)^n n^3$ *(iii)* $a_n=3n+5$

(iv) $a_n=\dfrac{n+1}{2n+5}$ *(v)* $a_n=\dfrac{1}{(2n-1)^2}$ *(vi)* $a_2=n+3$

(vii) $a_n=\dfrac{1}{3^n}$ *(viii)* $a_n=3n-5$ *(ix)* $a_n=(n+1)a_{n-1}, a_1=1$

Solution:-

(i) $a_n=n+3$

For n = 1, $a_1=1+3=4$

For n = 2, $a_2=2+3=5$

For n = 3, $a_3 = 3+3=6$

Thus the sequence is $a_1, a_2, a_3, ... = 4, 5, 6, ...$

(ii) $a_n = (-1)^n n^3$

For $n = 1$, $a_1 = (-1)^1 (1)^3 = -1$

For $n = 2$, $a_2 = (-1)^2 (2)^3 = 8$

For $n = 3$, $a_3 = (-1)^3 (3)^3 = -27$

Thus the sequence is $a_1, a_2, a_3, ... = -1, 8, -27, ...$

(iii) $a_n = 3n+5$

For $n = 1$, $a_1 = 3(1)+5 = 8$

For $n = 2$, $a_2 = 3(2)+5 = 11$

For $n = 3$, $a_3 = 3(3)+5 = 14$

Thus the sequence is $a_1, a_2, a_3, ... = 8, 11, 14, ...$

(iv) $a_n = \dfrac{n+1}{2n+5}$

For $n = 1$, $a_1 = \dfrac{1+1}{2(1)+5} = \dfrac{2}{7}$

For $n = 2$, $a_2 = \dfrac{2+1}{2(2)+5} = \dfrac{3}{9} = \dfrac{1}{3}$

For $n = 3$, $a_3 = \dfrac{3+1}{2(3)+5} = \dfrac{4}{11}$

Thus the sequence is

$a_1, a_2, a_3, ... = \dfrac{2}{7}, \dfrac{1}{3}, \dfrac{4}{11}, ...$

(v) $a_n = \dfrac{1}{(2n-1)^2}$

For $n = 1$, $a_1 = \dfrac{1}{[2(1)-1]^2} = 1$

For $n = 2$, $a_2 = \dfrac{1}{[2(2)-1]^2} = \dfrac{1}{9}$

For $n = 3$, $a_3 = \frac{1}{[2(3)-1]^2} = \frac{1}{25}$

Thus the sequence is $a_1, a_2, a_3, \ldots = 1, \frac{1}{9}, \frac{1}{25}, \ldots$

(vi) $a_n = n+3$

For $n = 1$, $a_1 = 1+3 = 4$

For $n = 2$, $a_2 = 2+3 = 5$

For $n = 3$, $a_3 = 3+3 = 6$

Thus the sequence is $a_1, a_2, a_3, \ldots = 4, 5, 6, \ldots$

(vii) $a_n = \frac{1}{3^n}$

For $n = 1$, $a_1 = \frac{1}{3^1} = \frac{1}{3}$

For $n = 2$, $a_2 = \frac{1}{3^2} = \frac{1}{9}$

For $n = 3$, $a_3 = \frac{1}{3^3} = \frac{1}{27}$

Thus the sequence is $a_1, a_2, a_3, \ldots = \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$

(viii) $a_n = 3n-5$

For $n = 1$, $a_1 = 3(1)-5 = -2$

For $n = 2$, $a_2 = 3(2)-5 = 1$

For $n = 3$, $a_3 = 3(3)-5 = 4$

Thus the sequence is $a_1, a_2, a_3, \ldots = -2, 1, 4, \ldots$

(ix) $a_n = (n+1)a_{n-1}$ $\quad a_1 = 1$

For $n = 2$, $a_2 = (2+1)a_{2-1} = 3a_1$

$a_2 = 3(1) = 3 \quad \because \quad a_1 = 1$

For $n = 3$, $a_3 = (3+1)a_{3-1} = 4a_2$

$a_3 = 4(3) = 12$

Thus the sequence is $a_1, a_2, a_3, \ldots = 1, 3, 12, \ldots$

Q.2- Find the terms indicated in the following sequences

(i) $2, 6, 11, 17 \ldots a_8$ **(ii)** $1, 3, 12, 60 \ldots a_7$

(iii) $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27} \ldots a_6$ **(iv)** $-1, 1, 3, 5, \ldots a_9$

(v) $\frac{1}{3}, \frac{2}{5}, \ldots a_5$ **(vi)** $1, -3, 5, -7 \ldots a_9$

Solution:-

(i) $2, 6, 11, 17, \ldots a_8 = ?$

Here we see that 4 is added to 1st term , 5 is added to 2nd term and 6 is added to 3rd term and so on.

Thus we get

$2, 6, 11, 17, 24, 32, 41, 51, \ldots$

Thus $a_8 = 51$ Ans.

(ii) $1, 3, 12, 60, \ldots a_7 = ?$

1st, 2nd Third terms are multipled by $3, 4$ 5 respectively to find the next term. Thus in this way we get

$1, 3, 12, 60, 360, 2520, 20160, \ldots$

Thus $a_7 = 20160$ Ans.

(iii) $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots a_6 = ?$

The given sequence is a G. P with Common ratio $\frac{1}{3}$

So we get

$1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{21}, \frac{1}{243}, \ldots\ldots\ldots$

Thus $a_6 = \frac{1}{243}$ Ans.

(iv) $-1, 1, 3, \ldots\ldots a_9 = ?$

It is an A. P with common difference of 2. So we get

$-1, 1, 3, 4, 5, 7, 9, 11, 13, 15, \ldots$

Thus $a_9 = 15$ Ans.

(v) $\frac{1}{3}, \frac{2}{5}, \ldots a_5 = ?$

The 1st two terms show that the sequence is

$\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}$

Thus $a_5 = \frac{5}{11}$ Ans.

(vi) $1, -3, 5, -7 \ldots a_9 = ?$

Thus study of four terms shows that the Sequence is

$1, -3, 5, -7, 9, -11, 13, -15, 17, \ldots$

Thus $a_9 = 17$ Ans.

Q.3- Find the next four terms of the following sequences

(i) $12, 16, 20, 27 \ldots$ **(ii)** $1, 3, 7, 15, 31, \ldots$

(iii) $-1, 2, 12, 40, \ldots$ **(iv)** $9, 11, 14, 17, 19, 22, \ldots$

(v) $4, 8, 12, 16, \ldots\ldots$ **(vi)** $-2, 0, 2, 4, 6, 8, 10, \ldots\ldots$

Solution:-

(i) $12, 16, 21, 27 \ldots$

4, 5, 6, are added to first, 2nd and 3rd terms, this way we get the sequence

$12, 16, 21, 27, 34, 42, 51, 61, \ldots$

(ii) $1, 3, 7, 15, 31 \ldots$

Study these terms and write the sequence. Multiply each term by 2 and add 1, to get next term.

$1, 3, 7, 15, 31, 63, 127, 255, 511, \ldots$

(iii) $-1, 2, 12, 40, \ldots$

1st term is multiplied by 2 and then 4 is added to have 2nd term.

2nd term is multiplied by 2 and then 8 is added to obtain 3rd term.

3rd term is multiplied by 2 and then 16 is added.

Similarly next term can be found we get the sequence.

$-1, 2, 12, 40, 112, 288, 704, 1664, \ldots$

(iv) $9, 11, 14, 17, 19, 22, \ldots$

By considering the given terms, we find that the sequence is:

$9, 11, 14, 17, 19, 22, 25, 27, 30, 33, \ldots$

(v) $4, 8, 12, 16, \ldots$

This ia an A. P with common difference 4. So we get the sequence

$4, 8, 12, 16, 20, 24, 28, 32, \ldots$

(vi) $-2, 0, 2, 4, 6, 8, 10, \ldots$

This is also an A. P with common difference of 2. So the sequence is

$-2, 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, \ldots$

EXERCISE 7.2

Q.1- Find the specified term of the following A.P

(i) $3, 7, 11, \ldots;$ **61st term** **(ii)** $-4, -7, -10 \ldots a_{19}$

(iii) $6, 4, 2, \ldots;$ **45th term** **(iv)** $9, 14, 19 \ldots a_{14}$

(v) $11, 6, 1 \ldots a_{18}$

Solution:-

(i) $3, 7, 11, \ldots,$ 61st term $= a_{61} = ?$

Here, $a = 3,\ d = 7 - 3 = 4,\ n = 61$

We know that $a_n = a + (n-1)d$

Put the value of a, d and n

$$a_{61} = 3 + (61-1)(4) = 3 + 240 = 243 \text{ Ans.}$$

(ii) $-4, -7, -10, \ldots,\ a_{19} = ?$

Here, $a = -4,\ d = -3,\ n = 19$

We know that

$$a_n = a + (n-1)d$$

$$a_{19} = -4 + (19-1)(-3)$$

$$= -4 + (18)(-3) = -4 - 54 \quad a_{19} = -58 \text{ Ans.}$$

(iii) $6, 4, 2, \ldots$, 45th term $= a_{45} = ?$

Here, $a = 6,\ d = -2,\ n = -45$

We know that $a_n = a + (n-1)d$

$a_{45} = 6 + (45-1)(-2)$

$a_{45} = 6 + (44)(-2) = 6 - 88 = -82$ Ans.

(iv) $9, 14, 19, \ldots = a_{14} = ?$

Here, $a = 9,\ d = 5,\ n = 14$

We know that $a_n = a + (n-1)d$

$a_{14} = 9 + (14-1)(5) = 9 + 65 = 74$ Ans.

(v) $11, 6, 1, \ldots = a_{18} = ?$

Here, $a = 11,\ d = -5,\ n = 18$

We know that $a_n = a + (n-1)d$

$a_{18} = 11 + (18-1)(-5)$

$= 11 + 17(-5) = 11 - 85 = -74$ Ans.

Q.2- Find the missing element using the formula of A.P

$\mathbf{a_n = a + (n-1)d}$

(i) $a = 2,\ a_n = 402,\ n = 26$

(ii) $a_n = 81,\ d = -3,\ n = 18$

(iii) $a = 5,\ a_n = 61\ \ n = 15$

(iv) $a = 16,\ a_n = 0\ \ d = -\frac{1}{4}$

(v) $a = 10,\ a_n = 400\ \ d = 5$

(vi) $a_n = 261\ \ d = 4,\ n = 18$

Solution:-

(i) $a = 2,\ a_n = 402\ \ n = 26$

Here, $d = ?$

Using formula $a_n = a + (n-1)d$

Put the values. $402 = 2 + (26-1)d$

$402 = 2 + (25)d$

$25d = 402 - 2 = 400$

$d = \frac{400}{25} = 16 \Rightarrow d = 16$ Ans.

(ii) $a_n = 81$ $d = -3$, $n = 18$

Here, $a = ?$, So

Use the formula $a_n = a + (n-1)d$

Put the values. $81 = a + (18 - 1)(-3)$

$$81 = a + (17) - 3$$

$$a = 81 + 51 = 132$$

$$a = 132 \text{ Ans.}$$

(iii) $a = 5$, $a_n = 61$ $n = 15$

Here, $d = ?$, So

Use the formula $a_n = a + (n-1)d$

Put the values. $61 = 5 + (15 - 1)d$

$$61 - 5 = 14d \Rightarrow 14d = 56$$

$$d = \frac{56}{14} = 4 \text{ Ans.}$$

(iv) $a = 16$, $a_n = 0$ $d = -\frac{1}{4}$ $n = ?$

Here, $n = ?$, So

Use the formula $a_n = a + (n-1)d$

Put the values. $0 = 16 + (n-1)\left(-\frac{1}{4}\right)$

$$\frac{1}{4}(n-1) = 16$$

$$n - 1 = 16 \times 4$$

$$n = 64 + 1 = 65 \text{ Ans.}$$

(v) $a = 10$, $a_n = 400$ $d = 5$, $n = ?$

Here, $n = ?$, So

Use the formula $a_n = a + (n-1)d$

Put the values. $400 = 10 + (n-1)5$

$$5(n-1) = 400 - 10$$

$$n - 1 = \frac{390}{5} \quad n = 78 + 1 = 79 \text{ Ans.}$$

(vi) $a = 261,\ d = 4, n = 18, a = ?$

Here, $n = ?$, So

Use the formula $a_n = a + (n-1)d$

Put the values. $261 = a + (18 - 1)4$

$$= a + 17\ (4)$$

$$a + 68 = 261$$

$$a = 261 - 68 = 193 \text{ Ans.}$$

Q.3- Find the 15th term of an A. P where the 3rd term is 8 and the common difference is $\frac{1}{3}$

Solution:- $a_{15} = ?,\ a_3 = 8,\ d = \frac{1}{3}$

Consider, $a_3 = 8$

$\Rightarrow \quad a + 2d = 8$

$\Rightarrow \quad \because a_n = a + (n-1)d$

$\Rightarrow \quad a + 2\left(\frac{1}{3}\right) = 8$

$\Rightarrow \quad a = 8 - \frac{2}{3}$

$\Rightarrow \quad a = \frac{22}{3}$

Now $a_{15} = a + 14d \qquad \because a_n = a + (n-1)d$

$$= \frac{22}{3} + 14\left(\frac{1}{3}\right)$$

$$= \frac{36}{3} = 12 \qquad a_{15} = 12 \text{ Ans.}$$

Q.4- Which term of an A.P 6, 2, –2, ... is –146?

Solution:- $a = 6,\ d = -4,\ a_n = -146$ and $n = ?$

Put the values in the formula.

$$a_n = a + (n-1)d$$

$$-146 = 6 + (n-1)(-4)$$

$$-146-6=-4(n-1)$$
$$-152=-4(n-1)$$
$$(n-1)=\frac{152}{4}$$
$$n=38+1=39 \text{ Ans.}$$

Q.5- Which term of an A.P 5, 2, −1, ... is −118?

Solution:-

$$a=5,\ d=-3,\ a_n=-118\ \ n=?$$

Put the values in the formula.

$$a_n=a+(n-1)d$$
$$-118=5+(n-1)(-3)$$
$$-118-5=-3(n-1)$$
$$3(n-1)=123$$
$$n-1=\frac{123}{3}$$
$$n=41+1=42 \text{ Ans.}$$

Q.6- How many terms are there in an A.P in which $a_1=a=11,\ a_n=68,\ d=3$

Solution:-

$$a=11,\ a_n=68,\ d=3, n=?$$

Put the values in the formula.

$$a_n=a+(n-1)d$$
$$68=11+(n-1)(3)$$
$$3(n-1)=68-11$$
$$n-1=\frac{57}{3}$$
$$n=19+1=20 \text{ Ans.}$$

Q.7- Find the 11^{th} term of an A. P $2-x, 3-2x, 4-3x, \ldots$

Solution:-

$$a_{11}=?,\ a=2-x,\ n=11,\ d=1-x$$
$$a_{11}=a+10d$$

$= 2 - x + 10(1 - x)$

$a_{11} = 12 - 11x$ Ans.

Q.8- Find the n^{th} term of an A. P where $a_{n-5} = 3n + 9$.

Solution:-

$a_{n-5} = 3n + 9$

To find a_n, replace n by $n + 5$

In this equation

$a_{n+5-5} = 3(n + 5) + 9$

$a_n = 3n + 15 + 9$

$a_n = 3n + 24$ Ans.

Q.9- Find the n^{th} term of an A. P $\left(\frac{3}{4}\right)^2, \left(\frac{3}{7}\right)^2, \left(\frac{3}{10}\right)^2, \ldots$

Solution:-

The given sequence is

$\left(\frac{3}{4}\right)^2, \left(\frac{3}{7}\right)^2, \left(\frac{3}{10}\right)^2, \ldots$

We see that only denominator is changing, so consider the sequence of denominators.

$4, 7, 10, \ldots$

Here $a = 4$, $d = 3$, $a_n = ?$

$a_2 = a + (n - 1)d$

Put the values of a and d

$a_n = 4 + (n - 1)(3) = 3n + 1$

Thus the nth term of given sequence is

$$= \left(\frac{3}{3n + 1}\right)^2 \text{ Ans.}$$

Q.10- If the n^{th} term of an A. P is $3n - 5$. Find the A.P.

Solution:-

$a_n = 3n - 5$

Put $n = 1, 2, 3, 4, \ldots$, We get

$a_1 = 3(1) - 5 = -2$

$a_2 = 3(2) - 5 = 1$

$a_3 = 3(3) - 5 = 4$

$a_4 = 3(4) - 5 = 7$

Thus the A. P. is

$-2, 1, 4, 7, \ldots$ Ans.

EXERCISE 7.3

Q.1- Find A.M between:

(i) $-3, 7$ **(ii)** $x-1, x+7$

(iii) $\sqrt{7}, 3\sqrt{7}$ **(iv)** $x^2+x+1; x^2-x+1$

Solution:-

(i) Here $a = -3, b = 7, A = ?$

$$A = \frac{a+b}{2} = \frac{-3+7}{2} \quad A = \frac{4}{2} = 2 \text{ Ans.}$$

(ii) Here $a = x-1, b = x+7, A = ?$

$$A = \frac{a+b}{2} = \frac{x-1+x+7}{2}$$

$$A = \frac{2x+6}{2} = \frac{2(x+3)}{2} = (x+3) \text{ Ans.}$$

(iii) $a = \sqrt{7}, b = 3\sqrt{7}, A = ?$

$$A = \frac{a+b}{2} = \frac{\sqrt{7}+3\sqrt{7}}{2} \quad A = \frac{4\sqrt{7}}{2} = 2\sqrt{7} \text{ Ans.}$$

(iv) $a = x^2+x+1, b = x^2-x+1, A = ?$

$$A = \frac{a+b}{2}$$

$$A = \frac{x^2+x+1+x^2-x+1}{2}$$

$$A = \frac{2x^2+2}{2} = \frac{2(x^2+1)}{2}$$

$$A = x^2+1 \text{ Ans.}$$

Q.2- If 3 and 6 are two A.Ms between a and b, find a and b.

Solution:-

As 3 and 6 are two A. Ms between a and b.

So a , 3, 6, b are in A.P.

$\Rightarrow \quad 3-a=6-3=b-6=$ Common difference

$\Rightarrow \quad 3-a=3$ and $b-6=3$

$\Rightarrow \quad a=0$ and $b=9$ Ans.

Q.3- Find three A. Ms between 11 and 19.

Solution:-

Let A_1, A_2, A_3 be three A.Ms between 11 and 19.

So, 11, A_1, A_2, A_3 ; 19 are in A.P.

and $a_1=11$, $a_5=19$, $d=?$

We have.

$a_5=a+4d \qquad \because a_n=a+(n-1)d$,

$\Rightarrow \quad 19=11+4d$

$\Rightarrow 4d=19-11=8$

$d=\dfrac{8}{4}=2$

Thus.

$A_1=11+d=11+2=13$

$A_2=A_1+d=13+2=15$.

$A_3=A_2+d=15+2=17$

Thus 13, 15 and 17 are A.Ms between 11 and 19.

Q.4- Find three A. Ms between and $\sqrt{2}$ and $6\sqrt{2}$.

Solution:-

Let A_1, A_2, A_3 be A.Ms between $\sqrt{2}$ and $6\sqrt{2}$, Then

$\sqrt{2}$, A_1, A_2, A_3, $6\sqrt{2}$ are in A.P

Here $a=\sqrt{2}$ and $a_5=6\sqrt{2}$, $d=?$

Now $a_5=a+4d$

$$\Rightarrow 6\sqrt{2} = \sqrt{2} + 4d$$

$$4d = 6\sqrt{2} - \sqrt{2} = 5\sqrt{2}$$

$$d = \frac{5}{4}\sqrt{2}$$

Thus $A_1 = a + d = \frac{\sqrt{2}}{1} + \frac{5\sqrt{2}}{4}$

$$A_1 = \frac{4\sqrt{2} + 5\sqrt{2}}{4} = \frac{9\sqrt{2}}{4}$$

$$A_2 = A_1 + d = \frac{9\sqrt{2}}{4} + \frac{5\sqrt{2}}{4}$$

$$A_2 = \frac{9\sqrt{7} + 5\sqrt{2}}{4} = \frac{14\sqrt{2}}{4}$$

$$A_2 = \frac{7\sqrt{2}}{2}$$

$$A_3 = A_2 + d = \frac{7\sqrt{2}}{2} + \frac{5\sqrt{2}}{4}$$

$$A_3 = \frac{14\sqrt{2} + 5\sqrt{2}}{4} = \frac{19\sqrt{2}}{4}$$

Thus $\frac{9\sqrt{2}}{4}, \frac{7\sqrt{2}}{2}, \frac{19\sqrt{2}}{4}$ are the required AM.s

Q.5- Find 6 A. Ms between 5 and 8.

Solution:-

Let $A_1, A_2, A_3, A_4, A_5, A_6$ be the six A.Ms between 5 and 8, So

$5\ A_1, A_2, A_3, A_4, A_5, A_6, 8$ are in A.P

Here $a = 5$, $a_8 = 8$, $d = ?$

We have $a_8 = a + 7d$

$$\Rightarrow 8 = 5 + 7d$$

$$\Rightarrow 7d = 3 \Rightarrow d = \frac{3}{7}$$

Here $A_1 = a + d = 5 + \frac{3}{7}$

$A_1 = \frac{38}{7}$

$A_2 = A_1 + d = \frac{38}{7} + \frac{3}{7} = \frac{41}{7}$

$A_3 = A_2 + d = \frac{41}{7} + \frac{3}{7} = \frac{44}{7}$

$A_4 = A_3 + d = \frac{44}{7} + \frac{3}{7} = \frac{47}{7}$

$A_5 = A_4 + d = \frac{47}{7} + \frac{3}{7} = \frac{50}{7}$

$A_6 = A_5 + d = \frac{50}{7} + \frac{3}{7} = \frac{53}{7}$

Thus $\frac{38}{7}, \frac{41}{7}, \frac{44}{7}, \frac{47}{7}, \frac{50}{7}$, and $\frac{53}{7}$ are six AM.s between 5 and 8.

Q.6- Find 7 A. Ms between 8 and 12.

Solution:-

Let $A_1, A_2, A_3, A_4, A_5, A_6, A_7$ are the seven A.Ms between 8 and 12

So

$8\ A_1, A_2, A_3, A_4, A_5, A_6, A_7\ 12$ are in A.P

Here $a = 8,\ a_9 = 12,\ d = ?$

We have $a_9 = a + 8d$

$\Rightarrow 12 = 8 + 8d \Rightarrow 8d = 4$

$\Rightarrow d = \frac{1}{2}$

Now $A_1 = a + d = 8 + \frac{1}{2} = \frac{17}{2}$

$$A_2 = A_1 + d = \frac{17}{2} + \frac{1}{2} = 9$$

$$A_3 = A_2 + d = 9 + \frac{1}{2} = \frac{19}{2}$$

$$A_4 = A_3 + d = \frac{19}{2} + \frac{1}{2} = 10$$

$$A_5 = A_4 + d = 10 + \frac{1}{2} = \frac{21}{2}$$

$$A_6 = A_5 + d = \frac{21}{2} + \frac{1}{2} = 11$$

$$A_7 = A_6 + d = 11 + \frac{1}{2} = \frac{23}{2}$$

Thus $\frac{17}{2}, 9, \frac{19}{2}, 10, \frac{21}{2}, 11, \frac{23}{2}$ are the seven AM.s between 8 and 12.

Q.7- If the A. Ms between 5 and b is 10, then find the value of b.

Solution:- As 10 is the A.M between 5 and b,

So, $5, 10, b$ are in A.P

$\Rightarrow 10 - 5 = b - 10 \Rightarrow b - 10 = 5$

$\Rightarrow b = 15$ Ans.

Q.8- If the A. Ms between a and 10 is 40, then find the value of a.

Solution:- As 40 is the A.M between a and 10,

So, $a, 40, 10$ are in A.P

$\Rightarrow 40 - a = 10 - 40$

$40 - a = -30 \Rightarrow a = 40 + 30 = 70 \quad \Rightarrow a = 70$ Ans.

Q.9- If the three A. Ms between a and b are 5, 9 and 13, find a and b.

Solution:- As $5, 9, 13,$ are three A.M between a and b,

So, $a, 5, 9, 13, b$ are in A.P

$\Rightarrow 5-a=9-5=13-9=b-13$

$\Rightarrow a=5-4=\pm 1 \Rightarrow a=1$ Ans.

Also $b-13=4 \Rightarrow b=17$ Ans.

EXERCISE 7.4

Q.1- Find the 7th term of a G.P 2, 8, 32, ...

Solution:- Given G.P is $2, 8, 32, \ldots$

Here $a=2, r=\frac{8}{2}=4, \quad n=7, \quad a_7=?$

We have the formula

$a_n=ar^{n-1}$

$\Rightarrow a_7=2(4)^{7-1}=2(4)^6=2(4096)$

$a_7=8192$ Ans.

Q.2- Find the 11^{th} term of a G.P 2, 6, 18, ...

Solution:- Given G.P is $2, 6, 18, \ldots$

Here $a=2, r=\frac{6}{2}=3, a_{11}=?, n=11$

So $a_n=ar^{n-1}$

$\Rightarrow a_{11}=2(3)^{11-1}=2(3)^{10}$

$a_{11}=2(59049)=118098$ Ans.

Q.3- Find the 6^{th} term of a G.P $-\frac{3}{2}, 3, -6 \ldots$

Solution:-

The given G.P is $-\frac{3}{2}, 3, -6, \ldots$

Here $a=-\frac{3}{2}, r=\frac{3}{\left(-\frac{3}{2}\right)}=-2, \; a_6=?, \; n=6$

We have. $a_n=ar^{n-1}$

$a_6=\frac{3}{2}(-2)^{6-1}=-\frac{3}{2}(-2)^5$

$$= -\frac{3}{2}(-32) = -3(-16)$$

$a_n = 48$ Ans.

Q.4- Find the 5^{th} term of a G.P $4, -12, 36 \ldots$

Solution:- Given G.P is $4, -12, 36, \ldots$

Here $a = 4, r = \frac{-12}{4} = -3,\ a_5 = ?\ ,\ n = 5$

We have $a_n = ar^{n-1}$

$\Rightarrow a_5 = 4(-3)^{5-1} = 4(-3)^4$

$a_5 = 4(81) \Rightarrow a_5 = 324$ Ans.

Q.5- Find the missing elements of the G.P:

(i) $\mathbf{r = 10, a_n = 100, a = 1}$

(ii) $\mathbf{a_n = 400, r = 2, a = 25}$

(iii) $\mathbf{a = 128,\ r = \frac{1}{2},\ a_n = \frac{1}{4}}$

Solution:-

(i) $a_n = 100, r = 10, a = 1, n = ?$

$a_n = ar^{n-1}$

$\Rightarrow 100 = 1(10)^{n-1} \Rightarrow (10)^{n-1} = (10)^2$

$\Rightarrow n - 1 = 2 \Rightarrow n = 3$ Ans.

(ii) $a_n = 400, r = 2, a = 25, n = ?$

$a_n = ar^{n-1}$

$\Rightarrow 400 = 25(2)^{n-1} \Rightarrow 2^{n-1} = \frac{400}{25} = 16$

$2^{n-1} = 2^4$

$\Rightarrow n - 1 = 4 \Rightarrow n = 5$ Ans.

(iii) $a = 128,\ r = \frac{1}{2}, a_n = \frac{1}{4},\ n = ?$

Here we have $a_n = ar^{n-1}$

$$\frac{1}{4} = 128\left(\frac{1}{2}\right)^{n-1}$$

$$\Rightarrow \left(\frac{1}{2}\right)^{n-1} = \frac{1}{4\times128} = \frac{1}{2^2\times2^7}$$

$$\Rightarrow \left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^{9} \Rightarrow n-1=9 \quad \Rightarrow n=10 \text{ Ans.}$$

Q.6- Find the 11th term of a G.P whose 5th term is 9 and common ratio is 2.

Solution:- Here $a_n = ?,\ a_5 = 9, r = 2.$

We have $a_n = a r^{n-1}$

$a_5 = a r^4$

$9 = a(2)^4 \Rightarrow 16a = 9$

$$\Rightarrow a = \frac{9}{16}$$

Now $a_{11} = ar^{10} = \frac{9}{16}(2)^{10}$

$$a_{11} = \frac{9}{(2)^4}\times(2)^{10} = \frac{9}{\cancel{(2)^4}}\times\cancel{(2)^4}\times(2)^6$$

$a_{11} = 9\times64 = 576$ Ans.

Q.7- Find the 13th term of a G.P whose 7th term is 25 and common ratio is 3.

Solution:- $a_{13} = ?,\ a_7 = 25, r = 3$

We have $a_n = a r^{n-1}$

$\Rightarrow a_7 = a r^6 \Rightarrow 25 = a(3)^6$

$$\Rightarrow 25 = 729a \Rightarrow a = \frac{25}{729}$$

Now $a_{13} = a r^{12} \Rightarrow a_{13} = \left(\frac{25}{729}\right)(3)^{12}$

$$a_{13} = \frac{25}{(3)^6}\times(3)^6\times(3)^6$$

$a_{13} = 25\times(3)^6 = 25\times729$

$a_{13} = 18225$ Ans.

Q.8- If a, b, c, d, are in G.P , show that, $a-b$, $b-c$, $c-d$ are in G.P.

Solution:- As a, b, c, d are in G.P

So $\frac{b}{a}=\frac{c}{b}=\frac{d}{c}=$ Common Ratio

$\Rightarrow \frac{b}{a}=\frac{c}{b}$, $\frac{c}{b}=\frac{d}{c}$ and $\frac{d}{c}=\frac{b}{a}$

$\Rightarrow b^2=ac$, $c^2=bd$ $ad=bc$ .... (A)

Now we have to Prove that

$a-b, b-c, c-d$ are in G.P

Consider

$(b-c)^2=b^2+c^2-2bc$

$=b^2+c^2-bc-bc$

Using results (A)

$(b-c)^2=ac+bd-ad-bc$

$=ac-ad-bc+bd$

$=a(c-d)-b(c-d)$

$(b-c)(b-c)=(a-d)(c-d)$

$\frac{(b-c)}{(a-b)}=\frac{(c-d)}{(b-c)}$

It means.

$a-b, b-c, c-d$ are in G.P

Q.9- Find the n^{th} term of a G.P, if $\frac{a_5}{a_3}=\frac{4}{9}$ and $a_2=\frac{4}{9}$

Solution:-

$a_n=?$, $\frac{a_5}{a_3}=\frac{4}{9}$, $a_2=\frac{4}{9}$

Consider

$\frac{a_5}{a_3}=\frac{4}{9}$ $\Rightarrow \frac{ar^4}{ar^2}=\frac{4}{9}$

$$\Rightarrow r^2 = \frac{4}{9} \Rightarrow r = \pm\frac{2}{3}$$

$$a_2 = \frac{4}{9} \Rightarrow ar = \frac{4}{9}$$

If $r = +\frac{2}{3} \Rightarrow a\left(\frac{2}{3}\right) = \frac{4}{9} \Rightarrow a = \frac{2}{3}$

If $r = -\frac{2}{3} \Rightarrow a\left(-\frac{2}{3}\right) = \frac{4}{9} \Rightarrow a = -\frac{2}{3}$

Now $a_n = ar^{n-1}$

If $r = \frac{2}{3}$, $a = \frac{2}{3}$, Then

$a_n = \frac{2}{3}\left(\frac{2}{3}\right)^{n-1} = \left(\frac{2}{3}\right)^n$ Ans.

If $r = -\frac{2}{3}$, $a = -\frac{2}{3}$, Then

$$a_n = -\frac{2}{3}\left(-\frac{2}{3}\right)^{n-1} = \left(-\frac{2}{3}\right)^n$$

Thus $a_n = \left(\frac{2}{3}\right)^n$ Or $a_n = \left(-\frac{2}{3}\right)^n$ Ans.

Q.10- Find three consecutive numbers in G.P, whose sum is 26 and their product is 216.

Solution:- Let the three required numbers be

$\frac{a}{r}, a, ar$ in G.P.

By the 1st condition

$\frac{a}{r} + a + ar = 26$ .... (1)

Now using 2nd condition

$$\left(\frac{a}{r}\right)(a)(ar) = 216$$

$a^3 = 6^3 \Rightarrow a = 6$

Put it in (1) $\frac{6}{r} + 6 + 6r = 26$

$\frac{6}{r} + 6r = 20$

$\frac{3}{r} + 3r = 10$

$\Rightarrow 3 + 3r^2 = 10r$

$\Rightarrow 3r^2 - 10r + 3 = 0$

$\Rightarrow 3r^2 - 9r - r + 3 = 0$

$\Rightarrow 3r(r-3) - 1(r-3) = 0$

$\Rightarrow (3r-1)(r-3) = 0$

$\Rightarrow 3r - 1 = 0$ or $r - 3 = 0$

$r = \frac{1}{3}$ or $r = 3$

Now if $r = \frac{1}{3}$ and $a = 6$

The required numbers in A.P are

$\frac{a}{r}, a, ar = \frac{1}{\frac{1}{3}}, 6, 6\left(\frac{1}{3}\right) = 18, 6, 2$

If $a = 6$ and $r = 3$, Then

$\frac{a}{r}, a, ar = \frac{6}{3}, 6, 6(3) = 2, 6, 18$

Thus the numbers are.

$18, 6, 2$ or $2, 6, 18$ Ans.

Q.11- Find the 30[th] term of a G.P $x, 1, \frac{1}{x}, \ldots$

Solution:-

$a_{30} = ?, \ a = x, r = \frac{1}{x}, \ n = 30$

$a_{30} = ar^{29}$

$a_{30} = x\left(\frac{1}{x}\right)^{29} = \left(\frac{1}{x}\right)^{28}$

$a_{30} = \frac{1}{x^{28}}$ Ans.

Q.12- Find the p^{th} term of a G.P $x, x^3, x^5, \ldots$

Solution:-

$a_p = ?,\ a = x,\ r = x^2,\ n = p$

We have $a_n = ar^{n-1}$

$\Rightarrow a_p = x(x^2)^{p-1}$

$\Rightarrow a_p = x\,x^{2p-2} \Rightarrow a_p = x^{2p-2+1}$

$a_p = x^{2p-1}$ Ans.

SOLVED EXERCISES

EXERCISE 7.5

Q.1- Find G.M between: *(i)* 9 and 5 *(ii)* 4 and 9 *(iii)* −2 and −8.

Solution:-

(i) $a = 9, b = 5$

G.M $= \pm\sqrt{ab}$

$= \pm\sqrt{9\times 5}$

G $= \pm 3\sqrt{5}$ Ans.

(ii) $a = 4\ b = 9,$

G.M $= \pm\sqrt{ab} = \pm\sqrt{4\times 9} = \pm 2\times 3$

G $= \pm 6$ Ans.

(iii) $a = -2,$ and $b = -8$

G.M $= \pm\sqrt{ab} = \pm\sqrt{(-2)\times(-8)}$

$= \pm\sqrt{16} = \pm 4$

G $= \pm 4$ Ans.

Q.2- Insert two G.Ms between: *(i)* 1 and 8 *(ii)* 3 and 81

Solution:-

(i) Let G_1, and G_2 be the two G.Ms between 1 and 8.

So, $1, G_1, G_2, 8$ are in G.P

Here $a = 1, a_4 = 8, r = ?$

We have $a_n = ar^{n-1}$

$\Rightarrow a_4 = ar^3$

$\Rightarrow 8 = 1(r^3)$ Putting values of a_4 and a

$\Rightarrow r^3 = 2^3 \Rightarrow r = 2$

Now $G_1 = ar = 1(2) = 2$

$G_2 = G_1 r = 2(2) = 4$

Thus 2 and 4 are two G.Ms between 1 and 8

(ii) Let G_1, and G_2 two G.Ms between 3 and 81. So,

$3, G_1, G_2, 81$ are in G.P

Here $a = 3, a_4 = 81, r = ?$

We have $a_n = ar^{n-1}$

$\Rightarrow a_4 = ar^3$

$\Rightarrow 81 = 3(r^3) \Rightarrow r^3 = 27$

$\Rightarrow r^3 = 3^3 \Rightarrow r = 3$

Now $G_1 = ar = 3(3) = 9$

$G_2 = G_1 r = 9(3) = 27$

Thus 9 and 27 are two G.Ms between 3 and 81

Q.3- Insert three G.Ms between: *(i)* 1 and 16 *(ii)* 2 and 32

Solution:-

(i) Let G_1, G_2, G_3 be three G.Ms between 1 and 16.

So, $1, G_1, G_2, G_3, 16$ are in G.P

Here $a = 1, a_5 = 16, r = ?$

We have $a_n = ar^{n-1}$

$\Rightarrow a_5 = ar^4$

$16 = 1(r^4) \Rightarrow (r^4) = 16$

$\Rightarrow r^4 = 2^4 \Rightarrow r = 2$

Now $G_1 = ar = 1(2) = 2$

$G_2 = G_1 r = 2(2) = 4$

$G_3 = G_2 r = 4(2) = 8$

Thus $2, 4, 8$ are three G.Ms between 1 and 16. Ans.

(ii) Let G_1, G_2, G_3 be three G.Ms between 2 and 32.

So; $2, G_1, G_2, G_3\ 32$ are in G.P

Here $a = 2, a_5 = 32, r = ?$

We have $a_n = ar^{n-1}$

$\Rightarrow a_5 = ar^4$

$32 = 2(r^4) \Rightarrow (r^4) = 16$

$\Rightarrow r^4 = 2^4 \Rightarrow r = 2$

Now $G_1 = ar = 2(2) = 4$

$G_2 = G_1 r = 4(2) = 8$

$G_3 = G_2 r = 8(2) = 16$

Thus $4, 8, 16$ are three G.Ms between 2 and 32.

Q.4- Insert four real geometric means between:3 and 96

Solution:-

Let G_1, G_2, G_3, G_4 be four G.Ms between 3 and 96

So, $3, G_1, G_2, G_3, G_4\ 96$ are in G.P

Here $a = 3, a_6 = 96, r = ?$

Now $a_n = ar^{n-1}$

$\Rightarrow a_6 = ar^5 \Rightarrow 96 = 3r^5$

$\Rightarrow r^5 = 32 \Rightarrow r^5 = 2^5$

$\Rightarrow r = 2$

Now $G_1 = ar = 3(2) = 6$

$G_2 = G_1 r = 6(2) = 12$

$G_3 = G_2 r = 12(2) = 24$

$G_4 = G_3 r = 24(2) = 48$

Thus $6, 12, 24, 48$ are four G.Ms between 3 and 96.

Q.5- The A.Ms between two numbers is 5 and their positive G.M is 4. Find the numbers.

Solution:-

Let a and be the required numbers. According to the given conditions

A.M = 5 and G.M = 4

$\Rightarrow \frac{a+b}{2}=5$, and $\sqrt{ab}=4$

$a+b=10$ ....(1) and $ab=16$.........(2)

From (1) $b=10-a$, Put in (2)

$a(10-a)=16$

$\Rightarrow 10a-a^2=16$

$\Rightarrow a^2-10a+16=0$

$\Rightarrow a^2-8a-2a+16=0$

$\Rightarrow a(a-8)-2(a-8)=0$

$\Rightarrow (a-2)(a-8)=0$

$\Rightarrow a-2=0$ or $a-8=0$

$a=2$ or $a=8$

Put these in (1), We get.

$b=8$ Or $b=2$ Ans.

Thus the required numbers are 2 and 8

Q.6- The positive G.M between two numbers is 6 and the A.M between them is 10. Find the numbers.

Solution:-

Let a and b be the two required numbers.

So, according to the given conditions

A.M = 10 and G.M = 6

$\Rightarrow \frac{a+b}{2}=10$ and $\sqrt{ab}=6$

$a+b=20$ ....(1) and $ab=36$ ....(2)

From (1) $b=20-a$, Put in (2) We get

$a(20-a)=36$

$20a-a^2=36$

$a^2-18a-2a+36=0$

$(a-2)(a-18)=0$

$\Rightarrow a-2=0$ or $a-18=0$

$a=2$ or $a=18$

Put these in (1), We get.

$b=18$ or $b=2$

Thus the required numbers are 2 and 18

Q.7- Show that the A.M between two numbers 4 and 8 is greater than their geometric mean.

Solution:-

$a=4$, $b=8$

$$\text{A.M}=\frac{a+b}{2}=\frac{4+8}{2}=6$$

$$\text{G.M}=\sqrt{ab}=\sqrt{4\times 8}=\sqrt{32}=5.66$$

Thus A.M > G.M $\because$ $6>5.66$

Q.8- Insert four geometric means between 160 and 5.

Solution:-

Let G_1, G_2, G_3, G_4 be four

G.Ms between 160 and 5

So, $160, G_1, G_2, G_3, G_4\ 5$ are in G.P

Here $a=160, a_6=5, r=?$

We have $a_6=ar^5$

$$\Rightarrow 5=160r^5 \Rightarrow r^5=\frac{5}{160}$$

$$\Rightarrow r^5=\frac{1}{32} \Rightarrow r^5=\left(\frac{1}{2}\right)^5$$

$$r=\frac{1}{2}$$

Thus $G_1 = ar = 160 \times \frac{1}{2} = 80$

$G_2 = G_1 r = 80 \times \frac{1}{2} = 40$

$G_3 = G_2 r = 40 \times \frac{1}{2} = 20$

$G_4 = G_3 r = 20 \times \frac{1}{2} = 10$

Thus $80, 40, 20, 10$ are four G.Ms between 160 and 5

Q.9- Insert three geometric means between 486 and 6.

Solution:-

Let G_1, G_2, G_3 be three G.Ms between 486 and 6

So, $486, G_1, G_2, G_3, 6$ are in G.P

Here $a = 486, a_5 = 6, r = ?$

We have $a_5 = ar^4$

$$\Rightarrow 6 = 486 r^4 \Rightarrow r^4 = \frac{4}{486} = \frac{1}{81}$$

$$\Rightarrow r^4 = \left(\frac{1}{3}\right)^4 \Rightarrow r = \frac{1}{3}$$

Thus $G_1 = ar = 486 \times \frac{1}{3} = 162$

$G_2 = G_1 r = 162 \times \frac{1}{3} = 54$

$G_3 = G_2 r = 54 \times \frac{1}{3} = 18$

Thus $162, 54, 18$ are three G.Ms between 486 and 6.

Q.10- Insert four geometric means between $\frac{1}{8}$ and 120.

Solution:- Let G_1, G_2, G_3, G_4 be four

G.Ms between $\frac{1}{8}$ and 120

So, $\frac{1}{8}, G_1, G_2, G_3, G_4\ 120$ are in G.P

Here $a=\frac{1}{8}, a_6 = 128, r = ?$

We have $a_6 = ar^5$

$$\Rightarrow 128 = \frac{1}{8}r^5 \Rightarrow r^5 = 1024$$

$$\Rightarrow r^5 = (4)^5 \Rightarrow r = 4$$

Thus $G_1 = ar = \frac{1}{8}\times 4 = \frac{1}{2}$

$$G_2 = G_1 r = \frac{1}{2}\times 4 = 2$$

$$G_3 = G_2 r = 2\times 4 = 8$$

$$G_4 = G_3 r = 8\times 4 = 32$$

Thus $\frac{1}{2}, 2, 8, 32$ are four G.Ms between $\frac{1}{8}$ and 128

Q.11- Insert six geometric means between 56 and $-\frac{7}{16}$.

Solution:-

Let $G_1, G_2, G_3, G_4, G_5, G_6$

be six G.Ms between 56 and $-\frac{7}{16}$

So, $56, G_1, G_2, G_3, G_4, G_5, G_6, -\frac{7}{16}$ are in G.P

Here $a = 56, a_8 = -\frac{7}{16}, r = ?$

We have $a_8 = ar^7 \Rightarrow -\frac{7}{16} = 56r^7$

$$\Rightarrow r^7 = -\frac{7}{16}\times\frac{1}{56}$$

$$\Rightarrow r^7 = -\frac{1}{128} \Rightarrow r^7 = \left(-\frac{1}{2}\right)^7$$

$$\Rightarrow r = -\frac{1}{2}$$

Thus $G_1 = ar = 56 \times -\frac{1}{2} = -28$

$$G_2 = G_1 r = -28 \times -\frac{1}{2} = 14$$

$$G_3 = G_2 r = 14 \times -\frac{1}{2} = -7$$

$$G_4 = G_3 r = -7 \times -\frac{1}{2} = \frac{7}{2}$$

$$G_5 = G_4 r = \frac{7}{2} \times -\frac{1}{2} = -\frac{7}{4}$$

$$G_6 = G_5 r = -\frac{7}{4} \times -\frac{1}{2} = \frac{7}{8}$$

Thus $-28, 14, -7, \frac{7}{2}, -\frac{7}{4}, \frac{7}{8}$ are four G.Ms between 56 and $-\frac{7}{16}$

Q.12- Insert five geometric means between $\frac{32}{81}$ and $\frac{9}{2}$

Solution:-

Let G_1, G_2, G_3, G_4, G_5

be five G.Ms between $\frac{32}{81}$ and $\frac{9}{2}$

So, $\frac{32}{81}, G_1, G_2, G_3, G_4, G_5, \frac{9}{2}$ are in G.P

Here $a = \frac{32}{81}, a_7 = \frac{9}{2}, r = ?$

We have $a_7 = ar^6 \Rightarrow \frac{9}{2} = \frac{32}{81}r^6$

$$\Rightarrow r^6 = \frac{9\times 81}{32\times 2} = \frac{729}{64}$$

$$\Rightarrow r^6 = \left(\frac{3}{2}\right)^6 \Rightarrow r = \frac{3}{2}$$

Now $G_1 = ar = \frac{32}{81}\times\frac{3}{2} = \frac{16}{27}$

$$G_2 = G_1 r = \frac{16}{27}\times\frac{3}{2} = \frac{8}{9}$$

$$G_3 = G_2 r = \frac{8}{9}\times\frac{3}{2} = \frac{4}{3}$$

$$G_4 = G_3 r = \frac{4}{3}\times\frac{3}{2} = 2$$

$$G_5 = G_4 r = 2\times\frac{3}{2} = 3$$

Thus $\frac{16}{27}, \frac{8}{9}, \frac{4}{3}, 2, 3$ are three G.Ms between $\frac{32}{81}$ and $\frac{9}{2}$.

Review Exercise 7

Q.1- Encircle the correct answer.

(i) Third term of $a_n = n+3$, when $n = 0$ is

(a) 3 (b) 6 (c) 9 (d) 0

(ii) Fourth term of $a_n = \frac{1}{(2n-1)^2}$, is

(a) $\frac{1}{7}$ (b) $\frac{1}{49}$ (c) $\frac{1}{81}$ (d) 0

(iii) For $2, 6, 11, 17, \ldots, a_5$ is

(a) 24 (b) 30 (c) 21 (d) 22

(iv) Next term of $12, 16, 21, 27$ is

(a) 34 (b) 30 (c) 31 (d) 32

(v) a_6 of $3, 7, 11, ...$ is

(a) 3 (b) 19 (c) 23 (d) 20

(vi) A.M between $\sqrt{3}$ and $3\sqrt{3}$ is

(a) $2\sqrt{3}$ (b) $5\sqrt{3}$ (c) $9\sqrt{3}$ (d) $4\sqrt{3}$

(vii) A.M between $2\sqrt{5}$ and $6\sqrt{5}$ is

(a) $4\sqrt{5}$ (b) $3\sqrt{5}$ (c) $5\sqrt{5}$ (d) $7\sqrt{5}$

(viii) a_5 of $2, 6, 18, ...$ is

(a) 160 (b) 161 (c) 162 (d) 30

(ix) G.M between -3 and -12 is

(a) ± 6 (b) 6 (c) -6 (d) ± 3

(x) G.M between 1 and 8 is

(a) $2\sqrt{2}$ (b) $\pm 2\sqrt{2}$ (c) $-2\sqrt{2}$ (d) $\sqrt{2}$

Ans:

(i) b	(ii) b	(iii) a	(iv) a
(v) b	(vi) a	(vii) a	(viii) c
(ix) c	(x) a		

Q.2- Fill in the blanks.

(i) The general or nth term of a sequence is denoted by___

(ii) If $a_n = 2n + 3$, then $a =$ _____

(iii) In an A.P $a_n = a + (n-1)d$, is called _____

(iv) A.M between 5 and 15 is_____

(v) If a, A, b is an A.P then A =_____

(vi) In a G.P, "r" is called______

(vii) In a G.P, a_n = ______

(viii) If a, G, b is a G.P, then G =______

(xi) Positive geometric mean between 2 and 3 is______

(x) The n^{th} term of an A.P when $a_{n-5} = 3n + 9$

Ans:

(i) a_n	(ii) 5	(iii) General term	(iv) 10
(v) $\frac{a+b}{2}$	(vi) Common ratio	(vii) ar^{n-1}	(viii) $\pm\sqrt{ab}$
(ix) $\sqrt{6}$	(x) $a_n = 3n + 24$		

Q.3- Find the general term and the 18th term of an A.P, whose first term is 3 and the common difference is 2.

Solution:- We are given that

$a = 3,\ d = 2, a_n = ?, a_{18} = ?$

Using the formula $a_n = a + (n-1)d$

Putting the values of a and d, We get

$a_n = 3 + (n-1)(2)$

$a_n = 3 + 2n - 2$

$a_n = 2n + 1$ Ans.

To find a_{18}, Put $n = 18$

$a_{18} = 2(18) + 1 = 37$ Ans.

Q.4- Find the n^{th} term of an A.P $\left(\frac{3}{5}\right)^3, \left(\frac{3}{7}\right)^3, + \left(\frac{3}{9}\right)^3, \ldots$

Solution:- Consider the sequence of denominates 5, 7, 9, ...

This is an A.P and.

Here $a = 5,\ d = 2, a_n = ?$

Using the formula $a_n = a + (n-1)d$

Putting the values of a and d, We get

$a_n = 5 + (n-1)(2)$

$a_n = 5 + 2n - 2$

$a_n = 2n + 3$

Thus the n^{th} term of given sequence is

$$a_n = \left(\frac{3}{2n+3}\right)^3$$

Q.5- If the A.M between *a* and 16 is 24. Then find the value of '*a*'.

Solution:- We are given that

A.M between a and $16 = 24$

$$\Rightarrow \frac{a+16}{2} = 24$$

$$a + 16 = 48$$

$$a = 48 - 16 = 32$$

$a = 32$ Ans.

Q.6- Find the 15th term of a G.P. whose 7th term is 27 and common ratio is 3.

Solution:- For a G.P $a_{15} = ?$

$a_7 = 27$, $r = 3$

$$\Rightarrow ar^6 = 27$$

$$a(3)^6 = 27$$

$$\Rightarrow a = \frac{27}{(3)^6} = \frac{27}{729} = \frac{1}{27}$$

Now $a_{15} = ar^{14}$

$$= \frac{1}{27}(3)^{14} = \frac{(3)^{14}}{(3)^3}$$

$$= (3)^{14-3} = 3^{11}$$

$a_{15} = 3^{11}$ Ans.

Q.7- Insert four Geometric Means between $\frac{1}{2}$ and 16.

Solution:-

Let G_1, G_2, G_3, G_4 be four G.Ms between $\frac{1}{2}$ and 16.

So, $\frac{1}{2}, G_1, G_2, G_3, G_4, 16$ are in G.P

Here, $a = \frac{1}{2}, a_6 = 16, r = ?$

We know that $a_6 = ar^5$

$16 = \frac{1}{2} r^5$

$r^5 = 32$

$r^5 = (2)^5 \Rightarrow r = 2$

Thus $G_1 = ar = \frac{1}{2} \times 2 = 1$

$G_2 = G_1 r = 1(2) = 2$

$G_3 = G_2 r = (2)(2) = 4$

$G_4 = G_3 r = (4)(2) = 8$

Thus $1, 2, 4, 8$ are four G.Ms between $\frac{1}{2}$ and 16.

Q.8- Find the three consecutive numbers in G.P, whose sum is 26 and their product is 216.

Solution:-

Let $\frac{a}{r}, a, ar$ be the required numbers in G.P. So

According to the given conditions.

$\frac{a}{r} + a + ar = 26$ ....(1)

And $\left(\frac{a}{r}\right)(a)(ar) = 216$

$a^3 = (6)^3$

$a = 6$

Put it in (1)

$\frac{6}{r} + 6 + 6r = 26$

$\frac{6}{r} + 6r = 20$

$\frac{3}{r} + 3r = 10$

$\Rightarrow 3+3r^2=10r$

$\Rightarrow 3r^2-10r+3=0$

$\Rightarrow 3r(r-3)-1(r-3)=0$

$\Rightarrow (3r-1)(r-3)=0$

$\Rightarrow 3r-1=0$ or $r-3=0$

$r=\frac{1}{3}$ or $r=3$

Thus if $a=6$ and $r=\frac{1}{3}$

the required numbers are

$\frac{a}{r}, a, ar$

$=\frac{6}{\frac{1}{3}}, 6, 6\left(\frac{1}{3}\right)$

$=18, 6, 2$

If $r=3$, $a=6$, Then

$\frac{a}{r}, a, ar=\frac{6}{3}, 6, 6(3)=2, 6, 18$

Thus $2, 6, 18$ are the required three numbers.

MULTIPLE CHOICE QUESTIONS

Q.1- Tick the Correct answer:

(i) If $2, 5, 9, 14, \ldots$ is a sequence then $7th$ term is

(a) 28 (b) 35

(c) 44 (d) 40

(ii) Given that $a_{n-2}=3n+2$, then $a_3=?$

(a) 11 (b) 13

(c) 15 (d) 17

(iii) $2, 6, 11, 17, \ldots$ $a_8=?$

(a) 41 (b) 51 (c) 31 (d) 32

(iv) In an A.P general term is

(a) $a+(n+1)d$ (b) $a+(n-1)d$

(c) $a-(n+1)d$ (d) $a-(n-1)d$

(v) In an A.P $a=-1$, $d=1$ then $a_n=?$

(a) n (b) $n-1$

(c) $n-2$ (d) $n+1$

(vi) $7th$ term of the sequence $\left(\frac{3}{7}\right)^2, \left(\frac{3}{10}\right)^2, \left(\frac{3}{13}\right)^2, \ldots$ is

(a) $\left(\frac{3}{19}\right)^2$ (b) $\left(\frac{3}{22}\right)^2$

(c) $\left(\frac{3}{25}\right)^2$ (d) $\left(\frac{3}{20}\right)^2$

(vii) Which term of the sequence $6, 2, -2, \ldots$ is -30

(a) $8th$ (b) $9th$ (c) $10th$ (d) $11th$

(viii) If 8 and 12 are two A.Ms between a and b The values of a and b are.

(a) $4,10$ (b) $4,16$ (c) $6,10$ (d) $10,14$

(ix) $6th$ term of G.P $2, 6, 18, \ldots$ is

(a) 162 (b) 486 (c) 1458 (d) 54

(x) A.M between x^2+x+1 and x^2-x+1 is

(a) x^2+1 (b) x^2-1 (c) $1-x^2$ (d) $2x^2+1$

(xi) The $30th$ term of G.P $x, 1, \frac{1}{x}, \ldots$ is

(a) x^{29} (b) x^{28} (c) $\frac{1}{x^{28}}$ (d) $\frac{1}{x^{30}}$

(xii) G.M between $2x^2$ and $8y^4$ is

(a) $\pm 5xy^2$ (b) $\pm 4xy^2$ (c) $\pm 4x^2y$ (d) $\pm 4x^2y^4$

(xiii) Two G.Ms between 4 and $\frac{1}{2}$ are.

(a) $2,1$ (b) $2,0$ (c) $3,1$ (d) $1, \frac{1}{4}$

(xiv) G.Ms between -2 and -8 is.

(a) -5 (b) -4 (c) $+4$ (d) ± 4

(xv) A.M between a and 16 is 24. Then $a=?$

(a) 8 (b) 32 (c) 10 (d) 30

(xvi) The basic Property of A.P is

(a) Common Ratio (b) Common Factor
(c) Common Difference (d) Common Divisor

(xvii) The basic Property of G.P is

(a) Common Ratio (b) Common Factor
(c) Common Difference (d) Common Divisor

MODEL CLASS TEST

Time : One Hour Max Marks : 25

Q.1- Tick the Correct answer. (7)

(i) A sequence having its last term is called

(a) Finite sequence (b) Infinite sequence
(c) Arithmetic sequence (d) G.P

(ii) $a_{n-2} = 5n-6$ Then a_4 is equal to

(a) 14 (b) 24 (c) 34 (d) 4

(iii) The sequence $\frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \ldots$

(a) Finite sequence (b) an A.P (c) G.P (d) H.P

(iv) A.M between $2\sqrt{5}$ and $6\sqrt{5}$ is

(a) $3\sqrt{5}$ (b) $4\sqrt{5}$
(c) $5\sqrt{5}$ (d) $5\sqrt{10}$

(v) The basic Property of G.P is

(a) Common Difference (b) Common Ratio
(c) Common Factor (d) Common Divisor

(vi) If a, G, b, are in G.P. Then G is called.

(a) Geometric Mean (b) Arithmetic mean
(c) Harmonic Mean (d) Mean

(vii) *nth* term of a sequence is $2n-7$
Then *20th* term is.
(a) 30 (b) 31 (c) 32 (d) 33

Q.2- Attempt any Five of the following short questions.

(i) Write the next three terms of sequence
$1, 9, 25, \ldots$

(ii) Find the general term of an A.P whose *1st* term is *2* and the common difference is *5*.

(iii) In an A.P , $a_1=3, d=4, a_n=59$
Find the number of terms.

(iv) If *3* and *6* are two A.Ms between a and b. Find a and b.

(v) Find the *pth* term of G.P $x, x^3, x^5, \ldots$

(vi) Insert two G.Ms between *4* and $\frac{1}{2}$

(vii) Find the *nth* term of sequence
$\left(\frac{3}{5}\right)^3, \left(\frac{3}{7}\right)^3, \left(\frac{3}{9}\right)^3, \ldots$

Q.3- Attempt any two questions of the following $2\times4=8$

(i) Find *15th* term of an A.P, where *3rd* term is *8* and the common difference is $\frac{1}{3}$

(ii) Insert four real G.Ms between *3* and *96*.

(iii) Insert three A.Ms between *11* and *19*.